

The generalized second law in the emergent universe

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Abstract

This paper studies whether the generalized second law of thermodynamics is fulfilled in the transition from a generic initial Einstein static phase to the inflationary phase, with constant Hubble rate, and from the end of the latter to the conventional era of thermal radiation dominated expansion. As it turns out, the said law is satisfied provided the radiation component does not largely contribute to the total energy of the static phase.

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I. INTRODUCTION

The highly successful hot big-bang model presents the uncomfortable feature of beginning the cosmic expansion out of an initial singularity, a state of infinity energy density and pressure where the laws of physics break down. To evade this serious shortcoming several solutions have been proposed. These include an ever bouncing universe (a universe that undergoes infinite cycles of expanding-contracting phases), and the so-called emergent universe.

The latter scenario replaces the initial singularity by an Einstein static phase in which the scale factor of the Friedmann-Robertson-Walker (FRW) metric does not vanish and, accordingly, the energy density, pressure and so on do not diverge. In the usual description the Universe starts expanding from the said phase, smoothly joins a stage of exponential inflation followed by standard reheating to subsequently approach the classical thermal radiation dominated era of the conventional big bang cosmology [1, 2]. Figure 1 summarizes this evolution.

This Letter studies which constraints (if any) the generalized second law (GSL) of thermodynamics imposes on the two intermediate phases, i.e., from the static phase to exponential inflation and from the reheating to thermal radiation domination (in the static, inflationary, and thermal radiation dominated phases the GSL is trivially fulfilled). According to the GSL the entropy of the horizon plus the entropy of the matter and fields within the horizon cannot decrease [3–5]. In the present case neither the particle horizon nor the event horizon exist in the static phase; only the apparent horizon does exist in all the phases dealt with here, therefore this is the one to be considered. Moreover, it has been reasoned that the latter horizon is the appropriate one in dealing with the second law of thermodynamics [6].

The apparent horizon in FRW universes is defined as the marginally trapped surface with vanishing expansion [7]; accordingly, its radius is given by $\tilde{r}_A = 1/\sqrt{H^2 + k a^{-2}}$, where $H = \dot{a}/a$ denotes the Hubble function and a the scale factor of the FRW metric (for reasons given below we just consider $k = +1$).

As is widely admitted, the entropy of the apparent horizon is not simply proportional to the horizon area but it includes quantum modifications,

$$S_A = k_B \left[\frac{\mathcal{A}}{4 \ell_{pl}^2} + \alpha \ln \frac{\mathcal{A}}{4 \ell_{pl}^2} \right]. \quad (1)$$

Here, k_B is Boltzmann's constant, ℓ_{pl} Planck's length, and α an $\mathcal{O}(1)$ constant. The first term is the classical one and varies directly as the area of the apparent horizon, $\mathcal{A} = 4\pi\tilde{r}_A^2$, while the second one arises from quantum corrections [8–11]. Therefore, in those situations in which the entropy inside the horizon can be ignored the second law of thermodynamics simply imposes that the constraint

$$S'_\mathcal{A} = k_B \left[\frac{1}{4\ell_{pl}^2} + \frac{\alpha}{\mathcal{A}} \right] \mathcal{A}' \geq 0 \quad (2)$$

must be satisfied at all times. Here $\mathcal{A}' = -(\mathcal{A}^2/2\pi)(HH' - k a^{-3})$ and the prime means derivative with respect to the scale factor.

Section II briefly summarizes the main features of the emergent scenario of Refs. [1] and [2]. Section III examines whether the GSL is satisfied in the transitions from the static phase to exponential inflationary phase and from the end of the latter to the expansion era dominated by thermal radiation. Finally, section IV summarizes our findings.

II. THE EMERGENT SCENARIO

In this scenario the Universe begins expanding from a finite initial size, say a_I , thus avoiding the singularity that lurks at the starting point of the standard big-bang model. Since one is free to choose $a_I \gg \ell_{pl}$ the quantum gravity era is also avoided. No exotic energy is needed. It suffices, e.g. ordinary matter (subscript m), radiation (subscript γ), and a quintessence scalar field ϕ with potential $V(\phi)$ obeying Klein-Gordon equation - actually, in the static phase matter and radiation are not strictly necessary but we know of no obvious reason to exclude them. The total energy density in this phase is simply $\rho_{m,I} + \rho_{\gamma,I} + (1/2)\dot{\phi}_I^2 + V_I = \frac{3k}{8\pi G a_I^2}$, which implies that this universe must have closed spatial sections (this is why we set $k = +1$).

To implement the scenario the potential must be asymptotically flat in the infinite past,

$$V(\phi) \rightarrow V_I \quad \text{as} \quad \phi \rightarrow -\infty, \quad t \rightarrow -\infty, \quad (3)$$

and fall toward a minimum at some finite value. As a consequence, the quintessence field rolls down from the static state at $-\infty$ and the potential slowly decreases from its initial value, V_I , in the infinite past. To have acceleration the inequality $V(\phi) - \dot{\phi}^2 > 0$ must be fulfilled. Since $V(\phi)$ decreases and $\dot{\phi}^2$ augments, at some time, say $t = t_e$, inflation terminates, then ϕ

oscillates about the minimum and reheating takes place, the latter followed by the radiation dominated era -see references [1] and [2] for details.

III. THE GSL AT THE TRANSITION PHASES

A. Transition from the static to the inflationary phase

This period corresponds to the scale factor interval $a_I < a < a_{inf}$ in Fig. 1.

During the static phase, $H_I = 0$, of this closed universe the scale factor, $a = a_I$, remains constant and so the total energy density, $\rho_I = 3k/(8\pi G a_I^2)$ [1]. Accordingly, the horizon, whose area reduces to $\mathcal{A}_I = 4\pi a_I^2$, exists just because the spatial curvature is positive.

In this transition $H' > 0$ but the sign of \mathcal{A}' rests also on the curvature and therefore on the relationship between H and the scale factor, i.e., it crucially depends on the specific model. As in the emergent scenario of [1] and [2], if the model conforms general relativity, we can write

$$H H' - \frac{k}{a^3} = -4\pi G \frac{\rho + p}{a}, \quad (4)$$

and

$$\mathcal{A}' = 2G \mathcal{A}^2 \frac{\rho + p}{a}, \quad (5)$$

whence the horizon area augments only when the overall gravity source respects the dominant energy condition. The compatibility of the latter with the increase of the Hubble factor might look counter-intuitive. Nevertheless, it can be realized -for instance- in universes whose expansion is dominated by matter and/or radiation and a quintessence scalar field. It is worthy of note that the positive spatial curvature makes possible both the Einstein static phase and the increase of the horizon area during this transition phase.

Still, $S'_\mathcal{A}$ could be negative if the square parenthesis on the right hand side of (2) were negative. However, for this to occur one should have $\alpha < -\mathcal{A}/(4\ell_{pl}^2)$. Since it is rather reasonable to expect $\ell_{pl}^2 \ll \mathcal{A}$ at all times, for the parenthesis to be negative α should take a very large negative value, something highly disfavored [12].

At this point one may wonder whether the entropy of the matter and/or radiation in the horizon volume may increase or decrease (we do not consider the entropy of the scalar field for we assume that the latter is in a pure quantum state). We first consider pressureless matter. At any given instant the entropy of dust inside the horizon obeys $S_m = k_B (4\pi/3) \tilde{r}_A^3 n$, where

$n = 3N/(4\pi a_I^3)$ is the number density of dust particles and N the total number particles in the static phase of radius a_I . In consequence,

$$S'_m = -3k_B \frac{N}{a_I^3} \tilde{r}_A^5 \left(H H' - \frac{k}{a^3} \right). \quad (6)$$

Again, if the model is governed by general relativity and complies with the dominant energy condition (i.e., $\rho + p > 0$), Eq. (4) ensures that S'_m will be positive-definite.

For radiation (as for any fluid possessing pressure) the entropy obeys Gibbs equation [13]. In our case,

$$T_\gamma S'_\gamma = \frac{d}{da} \left(\frac{4\pi}{3} \tilde{r}_A^3 \rho_\gamma \right) + w_\gamma \rho_\gamma \frac{d}{da} \left(\frac{4\pi}{3} \tilde{r}_A^3 \right) = 2\pi (1 + w_\gamma)(1 + 3w) \tilde{r}_A^3 \frac{\rho_\gamma}{a}. \quad (7)$$

In arriving to the second equality we have used $\rho'_\gamma = -3(1 + w_\gamma) \rho_\gamma/a$ and Eq. (4). Because of the long duration of the static phase we can safely assume that the radiation is thermalized (i.e., it presents a black-body distribution) thereby we can set $w_\gamma = 1/3$. Since the expansion is accelerated the equation of state of the overall fluid (including the scalar field), $w = p/\rho$, must fulfill $1 + 3w < 0$ whence S'_γ results negative. In consequence, for the GSL to be satisfied the radiation component must not largely contribute to the total energy density. To be more specific, the GSL condition $S'_A + S'_m + S'_\gamma \geq 0$ translates into the constraint

$$\frac{\rho_\gamma}{\rho} \leq \frac{3}{4} \frac{k_B G \pi (1 + w) T_{\gamma I} a_I \tilde{r}_A \left[\frac{4}{\ell_{pl}^2} + 6 \frac{N}{a_I^3} \tilde{r}_A \right]}{|1 + 3w| a}. \quad (8)$$

In writing last equation we have neglected the second term in the square parenthesis in Eq. (2) and made use of the evolution law for the radiation temperature $T_\gamma = T_{\gamma I}(a_I/a)$, where $T_{\gamma I}$ stands for its value at the Einstein static phase. Since the ratio $\rho_\gamma/\rho \propto a^{3w}$ (with $w < 0$) decreases with expansion, the maximum value of the right hand side of (8) occurs when $a = a_I$. This implies the upper bound

$$\frac{\rho_\gamma}{\rho} \leq \frac{3}{4} \frac{k_B G \pi (1 + w) T_{\gamma I} a_I \left[\frac{4}{\ell_{pl}^2} + 6 \frac{N}{a_I^3} \right]}{|1 + 3w|}, \quad (9)$$

which must be respected if the GSL is to be fulfilled.

B. Transition between the exponential inflation regime and the thermal radiation dominated phase

This period starts at $a = a_e$ and terminates when the products of the inflaton decay get fully thermalized.

The exponential inflationary period is characterized by $H = \text{constant}$. At the end of it -according to cold inflation- a very fast phase of reheating takes place. In the course of the latter all the energy of the inflaton field is very quickly converted into a mixture of radiation and relativistic particles that begins dominating the expansion. Automatically, the strong energy condition, $\rho + p > 0$, is met and the Universe starts decelerating with $H' < 0$. However, since the mixture of radiation and particles produced by the decay of the inflaton takes some time (by all accounts much larger than the exponential inflation stage) in thermalize, the conventional thermal radiation dominated era (in which $w_\gamma = 1/3$) cannot commence right away, it has to wait. Here we analyze whether the GSL is fulfilled during the period connecting the reheating phase with the conventional thermal dominated phase.

Again \mathcal{A}' is given by Eq. (5) where now $\rho = \rho_\gamma$ and $p = p_\gamma$, whence $S'_\mathcal{A} > 0$. Likewise the entropy variation of the mixture of relativistic particles and radiation (of total energy ρ_γ) is given by the right hand side of Eq. (7) with w and w_γ replaced by \tilde{w} , the latter being positive-definite (the tilde is to remind us that the mixture is not thermalized though, obviously, $\tilde{w}_\gamma \rightarrow w_\gamma = 1/3$ as the thermalization proceeds). Thus, the GSL is guaranteed in this transition.

We have not considered dissipative processes like the decay of heavy (but still relativistic) particles into lighter ones and bulk viscosities. However, these processes being irreversible in nature necessarily augments the phase space and, accordingly, the entropy within the horizon.

IV. CONCLUDING REMARKS

A successful cosmological scenario is expected to fulfill the laws of thermodynamics, in particular the GSL. In this paper we found that the condition for the emergent scenario of Refs. [1] and [2] to comply with the GSL is that the radiation component do not contribute largely to the total energy density of the static phase. More precisely, that the bound (9) be met at the commencement of the transition from the static phase to the period of exponential inflation where $H = \text{constant}$. Most emergent scenarios are expected to easily fulfill this condition since for the Universe to transit from the static phase ($H = H_I = 0$) to the said period its expansion must be dominated by some energy component that violates the strong energy condition -as for instance a quintessence scalar field. It should be noted that

although neither radiation nor pressureless matter are necessary ingredients of the energy budget in the Einstein static phase it is quite natural to consider them. Clearly if radiation were absent, the GSL would be more readily satisfied.

We have ignored the exponential inflationary phase (i.e., the interval running from a_{inf} to a_e) because solely thermal radiation could contribute negatively to the total S' in this period. Such contribution, being proportional to ρ_γ (see Eq. (7)), is -in any case- vanishing since, due to the huge expansion from a_i to a_e in the emergent model under consideration, this component gets practically redshifted away before the onset of the said period.

While we focused on one specific scenario it encapsulates the main features any emergent model must possess. This is why we believe the overall result of this research should bear a wide generality and, in any case, it may serve as a springboard to the study of the GSL in more sophisticated emergent scenarios.

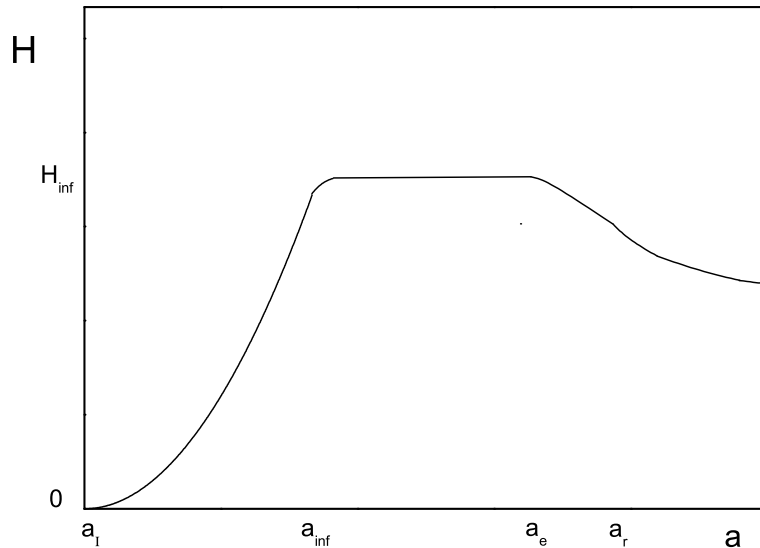


Figure 1. Schematic evolution of the Hubble function from the Einstein static era to the thermal radiation era. Here a_{inf} and a_e stand for the scale factor at the beginning and end of exponential inflation, respectively; a_r denotes the scale factor at some generic point at the radiation dominated expansion era where $w = \tilde{w}_\gamma$.

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